(2)

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1

1b. RESTRICTIVE MARKINGS

2

AD-A215 466

2

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)
AFOSR-TR- 89-1304

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
New York City Univ.		Air Force Office of Scientific Research

6c. ADDRESS (City, State, and ZIP Code)	7b. ADDRESS (City, State, and ZIP Code)
Department of Mathematics New York NY 10031	Building 410 Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING / SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	AFOSR-84-0095

8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.
Building 410 Bolling AFB, DC 20332-6448	61102F	2304	A5	

11. TITLE (Include Security Classification)
On the Reliability of Systems Subject to Maintenance and Repair

12. PERSONAL AUTHOR(S)
Mark Brown

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 1 Apr 84 TO 30 Sep 88	December 1988	7

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Several research results concerning reliability of systems to repair were obtained. These include inequalities for distributions with increasing failure rate identifying coefficients in the spectral representation for first passage times, and obtaining error bounds for exponential approximations of geometric distributions.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
Dr. Lajos Horvath	(202) 767-4940	NM

DD Form 1473, JUN 86 *Previous editions are obsolete.*

Final Scientific Report to Air Force
Office of Scientific Research

Grant AFOSR 84-0095

On the Reliability of Systems Subject to Maintenance and Repair

Department of Mathematics
The City College, CUNY
New York, NY 10031

Mark Brown, Principal Investigator

December 1988

Grant AFOSR-84-0095 was awarded for the period 4/1/84-3/31/85. It was subsequently extended to 6/30/85. Renewal proposals covering 7/1/85-6/30/86, 7/1/86-6/30/87 and 7/1/87-6/30/88, were submitted and approved. This report will discuss the research accomplishments achieved during the grant period.

1) A technical report, "Inequalities for distributions with increasing failure fate," was issued in December 1984 (City College, CUNY Report No. MB84-01). It was subsequently published in Contributions to the Theory and Application of Statistics. A Volume in Honor of Herbert Solomon, Academic Press, 3-17.

The main result is that if F is an IFR (increasing failure rate) distribution on $[0,\infty)$ with $\mu_i = \int x^i dF(x)$, $i=1,2$ assumed finite and $\rho = 1-(\mu_2/2\mu^2)$ then:

$$\sup_B \left| F(B) - \mu^{-1} \int_B e^{-t\mu^{-1}} dt \right| \leq 2\rho$$

where the sup is taken over all Borel sets in $[0,\infty)$. Thus an IFR distribution with small ρ is approximately exponential.

This result received commentary from Professors Richard Barlow and Ilya Gertsbakh in the November 1987 issue of Statistical Science. Both were discussants for a survey paper "Soviet Work in Reliability" by Rukhin and Hsieh. Barlow remarked that work was "excellent" and overlaps the work of the Gnedenko school of reliability at Moscow University. Gertsbakh remarks that the work considerably generalizes and improves on work of Soviet authors mentioned in the survey. Barlow and Gertsbakh both also cite the paper "Exponential Approximations for two Classes of Aging Distributions," by myself and G. Ge (Ann. Probab. 12, 869-875, 1984). This work was performed under an earlier AFOSR grant.

2) A technical report, "Identifying Coefficients in the Spectral Representation for First Passage Times," was issued in May 1986 (City College, CUNY Report MB84-02). It was subsequently published in Probability in the Engineering and Informational Sciences, 1987, 69-74.

The spectral approach to first passage time distributions for Markov processes requires knowledge of the eigenvectors and eigenvalues of the infinitesimal generator matrix. This paper demonstrates that in many cases knowledge of the eigenvalues alone is sufficient. This result simplifies computation and thus renders the spectral approach more attractive.

3) A technical report "Error Bounds for Exponential Approximations of Geometric Distributions," was issued in August 1986 (City College, CUNY Report MB84-03).

In it I solve a problem cited by Gertsbakh (1984) as being of "great interest for engineering applications." The applications include reliability computations for repairable systems as well as for systems with standby replacements. Additional applications include approximation of the waiting time distribution in G/G/1 queues, the approximate distribution of the waiting time to rare events in regenerative processes, and the approximate distribution for the time until the stress caused by random loads exceeds a given level. The topic of geometric convolutions has been studied by several Soviet authors as surveyed by Gertsbakh (1984). The idea is that geometric sums of non-negative i.i.d. random variables are approximately exponentially distributed. error bounds are required for use in applications. My bounds offer significant improvement over those currently available, and are shown to be close to the best possible.

4) A technical report "On a Correlation Inequality and its Applications," (City College, CUNY Report No. MB84-04) was issued in March 1988. It will appear in the conference proceedings of the Conference on Notions of Dependency in Probability and Statistics.

A correlation inequality is derived for the correlation between a non-negative random variable, X, and the cumulative hazard function H(X), where $H(t) = -\ell n(Pr(X > t))$. This inequality has several applications which are explored in the paper:

(i) The inequality of Brown (1984) discussed in (1) above is extended from IFR to DMRL (decreasing mean residual life) distributions.

(ii) The asymptotic savings in risk between two Monte Carlo estimators of the renewal function (Brown, Solomon and Stephens (1981)) is bounded in terms of the first two moments for NBUE distributions.

(iii) Bounds are obtained for the expected waiting time between the first and second record values corresponding to an i.i.d. sequence. This problem naturally arises in the study of maintenance policies. Inequalities are also obtained for the moments of higher record values.

(iv) It is shown that among all distributions with failure rate uniformly bounded above by λ, the exponential distribution with parameter λ has the minimum variance for the k^{th} record value for $k \geq 1$.

5) During the grant period I derived several results which will appear as part of technical reports to be issued under my current AFOSR grant. These included:

(i) Error bounds for exponential approximations. These bounds apply to weighted sup norms and allow for two sided bounds whose length goes to zero as $t > \omega$.

(ii) Age-weighted distributions. The age weighted distribution corresponding to a distribution F on $[0,\infty)$ is defined by $d(AF)(t) = t\mu^{-1}dF(t)$, where μ is the mean corresponding to F. Age weighted distributions arise naturally in several applications including renewal theory and Neyman-Pearson tests. I have systematically developed the properties of this transformation, obtaining some interesting results and applications.

(iii) Semi-Markov processes. I have investigated properties of the fundamental matrix applied to semi-Markov processes. Even though spectral representations are not available (as in Markov processes) the fundamental matrix is shown to be useful and computationally feasible.

Bibliography

I. Technical reports issued under Grant AFOSR-84-0095.

[1] "Inequalities for distributions with increasing failure rate, by Mark Brown," December 1984. City College, CUNY Report No. MB84-01, AFOSR Technical Report No. 84-01.

[2] "Identifying coefficients in the spectral representation for first passage times," by Mark Brown and Yi-Shi Shao, May 1986. City College, CUNY Report No. MB84-02, AFOSR Technical Report No. 84-02.

[3] "Error bounds for exponential approximations of geometric distributions," by Mark Brown, August 1986. City College, CUNY Report No. MB84-03, AFOSR Technical Report No. 84-03.

[4] "On a correlation inequality and its applications," by Mark Brown, March 1988. City College, CUNY Report No. MB84-04, AFOSR Technical Report No. 84-04.

II. Papers of Mark Brown published during the grant period including those supported under previous AFOSR grants.

[1] "On the choice of variance for the log rank test." Biometrika, 71 (1984), 65-74.

[2] "On the relibability of repairable systems." Operations Research, 32 (1984), 607-615.

[3] "Proximity between distributions: An inequality and its applications." Reliability Theory and Models, Academic Press, New York, 257-266 (1984).

[4] "On the waiting time for the first occurrence of a pattern," with G. Ge. Relibaility Theory and Models, Academic Press, New York, 267-272.

[5] "A measure of variability based on the harmonic mean and its use in approximations." The Annals of Statistics, 13, 1239-1243 (1985).

[6] "Inequalities for distributions with increasing failure rate." Contributions to the Theory and Application of Statistics. A Volume in Honor of Herbert Solomon. Academic Press, New York, 3-17.

[7] "Identifying coefficients in the spectral representation for first passage time distributions," with Yi-Shi Shao. Probability in the Engineering and Information Sciences, 1 (1987), 69-74.

III. Other papers referenced in this report.

[1] Gertshakh, I.B. (1984). "Asymptotic methods in reliability theory: A review." Adv. in Appl. Probab., 16, 147-175.

[2] Rukhin, A.L. and Hsieh, H.K. (1987). "Survey of Soviet work in reliability." Statistical Science, 2, 484-503.